

## Helping Students to Recognize and Evaluate an Assumption in Quantitative Reasoning: A Basic Critical-Thinking Activity with Marbles and Electronic Balance

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### Abstract

There is a general agreement that critical thinking is an important element of 21st century skills. Although critical thinking is a very complex and controversial conception, many would accept that recognition and evaluation of assumptions is a basic critical-thinking process. When students use simple mathematical model to reason quantitatively about a situation, they usually do not consider which implicit assumptions they have made and, consequently, they do not evaluate if these assumptions are acceptable in the related problem task. In order to show to students the importance of being aware of assumptions making and their consequences in the quality of judgments, a simple *Predict – Observe - Explain* active-learning sequence was designed. In it students are supposed (a) to observe the mass measurement of 10 marbles on an electronic balance and (b) to predict the future balance readings related to 7 and 12 marbles. They would be also asked explicitly (c) to state all assumptions they had made in order to predict the readings and (d) to propose a method to verify assumptions' acceptability. After they have observed actual readings, they would be asked to explain the differences and verify their explanations. Informal pilot tries of this learning sequence was carried out by a few high-school physics teachers during their regular classroom sessions and they considered that it gave good results with students.

**Keywords:** Critical thinking, science education, critical thinking, problem solving, physics activities.

### Introduction

There is a general agreement that critical thinking and problem solving are among highly-discussed “*21st century skills*” (Wagner, 2008; Trilling & Fadel, 2009; Wagner, 2010). Critical thinking is a rather complex construct that is differently conceptualized in historical, philosophical and cultural studies. For example, Moore (2013) has identified seven definitional strands related to critical thinking: (1) as judgment; (2) as skepticism; (3) as a simple originality; (4) as sensitive readings; (5) as rationality; (6) as an activist engagement with knowledge; and (7) as self-reflexivity.

Such complexity is partially revealed in a definition which was agreed by an expert forum, according to which critical thinking is “purposeful, self- regulatory judgment which results in interpretation, analysis, evaluation, and inference, as well as explanation of the evidential, conceptual, methodological, criteriological or contextual considerations upon which judgment is based” (Facione, 1990).

Different authors stress different aspects and parts of the critical-thinking construct. Simpson and Courtney (2002) point out that critical thinking processes require active argumentation, initiative, reasoning, envisioning, analyzing complex alternatives, and making contingency-related value judgments, while for Banning (2006), critical thinking involves scrutinizing, differentiating and appraising information as well as reflecting on the information that will be used to make judgments and inform clinical decisions. According to Brookfield (1987), essential for critical thinking are identifying, challenging, and analyzing assumptions for validity.

Black (2012) considers that basic critical-thinking processes are:

- (a) Analyzing arguments;
- (b) Judging the relevance and significance of information;
- (c) Constructing clear and coherent arguments; and
- (d) Forming well-reasoned judgments and decisions.

In addition, critical thinking requires an open-minded yet critical approach to one's own thinking as well as that of others.

Taking into account all above, it is possible to conclude that identification and evaluation of assumptions built in arguments, judgments and decisions is an important element of critical thinking.

### **Teaching critical-thinking skills**

Being critical-thinking skills so important educational goal, there are many different pedagogical approaches to teach them, either in special courses or in traditional courses. In order to enhance students' critical thinking in an undergraduate general science course, Sharma, Landa and Furlong (2013) designed and implemented active learning modules by incorporating group-based learning with authentic tasks, scaffolding, and individual reports. The results showed that these active learning strategies were useful.

Zhou and collaborators (2012) designed, implemented and evaluated a WebQuest teaching approach for chemistry classroom teaching in improving the critical thinking of high school students. Their results show that integrating WebQuest into science classroom teaching might be an effective way to develop high school students' critical thinking. Malamitsa, Kasoutas and Kokkotas (2009) explored how the integration of aspects of History of Science into instruction affects the development of sixth grade students' critical thinking skills in science courses. With that goal on mind, they designed and implemented a project on electromagnetism to engage primary school students in a critical examination of knowledge (importance, complexity and human implications) by generating argumentation and discussion in their classrooms. Their encouraging results regarding critical thinking skills development are supportive to the integration of aspects of History of Science in science.

Although many approaches to teaching critical thinking skills in different courses were tried in research format, the obtained, at least at the college level, results aren't conclusive regarding which is the most promising instructional intervention (Niu, Behar-Horenstein and Garvan, 2013).

### **Critical thinking in physics teaching**

While "problem solving" has 794 hits in publications of American Institute of Physics, "critical thinking" has been mentioned only in 28 articles. This bibliographic fact is surprising because the first explicit mention of "critical thinking" in *American Journal of Physics* was done long time ago. Namely, following then dominant conductivism paradigm, in the article "Testing for Critical Thinking in Physics", Burke (1949) has defined 15 behaviors physics students should exhibit in order to be considered as critical thinkers. He also argued that some items, which allegedly tested critical thinking, were testing other things. What is really strange is that the Burke's wasn't cited

in any of a few articles published in *American Journal of Physics* or *The Physics Teacher*, which mention “critical thinking”.

For developing critical thinking skills, Harvie (1987) proposed “comparison problems”. In these problems students were asked questions instead of being told what to calculate.

Rosen (2008) considered that the element of surprise in lab activities offers the opportunity to students to cultivate critical thinking skills. The surprise comes from the differences between textbooks treatment of collisions (perfect elastic collision, rotation-free collision) and collision features in real-world lab. When students observe these differences, they are ready to participate in a meaningful scientific discussion.

Paetkau (2007) considered that critical thinking may be emphasized at the end of a calculation involved in problem solving, by teaching students to ask routinely the question: does this answer make sense? To answer this question, they should compare the answer to what they know about the world. In other words, they should compare experiment (experience) with theory (calculation). Calculations that appear to contradict “common knowledge” help emphasize critical thinking.

In the example used by Paetkau (2007), heat loss from human head via conduction through a woolen hat was calculated and compared with calculated heat loss from bare head through radiation. The results of these plausible (but wrong) models of heat transfer (30 W for conductive loss and 20 W for radiative loss) contradict what they know from experience (and from what their mothers have told them). This disagreement should lead them to ask where they have gone wrong in the calculations, or they may turn their attention to the estimated numbers. Finally, they may question the models, which were used.

Along the similar path, Warren (2010) designed “critical thinking task” as those in which the students are given a problem and proposed solution, and asked to give three independent arguments which each analyze whether the given solution is reasonable. There are no explicit prompts to use any particular strategy, so the students must spontaneously recognize that special-case and unit analysis can each be used to generate arguments, and then use these strategies to make valid and sound arguments.

More complex conception of critical thinking in physics learning was used by Sulaiman (2011). According to him, elements of critical thinking are:

- (1) Making an inference;
- (2) Making an assumption;
- (3) Deduction;
- (4) Making an interpretation;
- (5) Evaluation argument.

Sulaiman (2011) investigated how problem-based learning (PBL) online and lecture-based learning (LBL) online affect critical thinking. He found, at general level, no significant differences between two groups. Nevertheless, at the levels of specific tasks, it was possible to identify statistically significant differences for making an inference (in favor of the PBL group) and assumption (in favor of the LBL group). These results of Sulaiman thesis contradicts well spread beliefs that problem-based learning necessary improves critical thinking skills (Kek & Huijser, 2011).

## Predicting Balance Readings with Different Number of Marbles

Predict – Observe – Explain (White & Gunstone, 1992) is a very useful learning sequence, which helps students

- (1) learn about the ideas they have about physical world and
- (2) test them against the reality.

One problem students have in formulating their predictions is to state which assumptions they make and to learn how significantly the predictions depend on the assumptions which have been made either explicitly or implicitly.

To help students gain an initial experience in both aspects of learning and to practice a basic critical-thinking process, we designed a simple activity with marbles and electronic balance. That activity was included in a high-school physics textbook (Slisko, 2010). During various workshops was possible to discuss its potential with high-school teachers who carried out it with their students.

The activity starts with an observation in which students see ten marbles on an electronic balance (**Figure 1**). They are told that the reading of the balance (64.124 g) represents the total mass of the marbles.



*Figure 1. Ten marbles on the electronic balance.*

Then the students are asked to

- (1) predict the reading of the balance with twelve and seven marbles on it;
- (2) state explicitly which assumption should be made in order that the reasoning is valid; and
- (3) how could that assumption be checked experimentally?

According to physics teachers, the students are much better in the first than in the second and third task. Some students find the “mass of one marble” dividing the mass of the 10 marbles by 10:

$$m_{\text{one marble}} = \frac{64.124 \text{ g}}{10} = 6.4124 \text{ g}.$$

Then, to find the balance reading for 12 and 7 marbles, they use the “mass of one marble” with 12 and 7:

$$m_{12 \text{ marbles}} = 12 m_{\text{one marble}} = 76.9488 \text{ g}.$$

$$m_{7 \text{ marbles}} = 7 m_{\text{one marble}} = 44.8868 \text{ g.}$$

More students use the “rule of three” to find the readings for 12 and 7 marbles:

$$\frac{x_{12}}{12} = \frac{64.124 \text{ g}}{10}$$

$$\frac{x_7}{7} = \frac{64.124 \text{ g}}{10},$$

where  $x_{12}$  and  $x_7$  are the unknown readings (the total masses of 12 and 7 marbles).

Students usually do not pay attention that balance can show the mass only up to one milligram. Being so, they do not round up the result of calculation (76.949 g and 44.887 g).

Nevertheless, more disturbing result is that only a few students are able to recognized that both procedure are based on the assumption that every marble has the same mass and that the veracity of the assumption can be checked by weighing every marble.

### Observing and explaining balance readings with different number of marbles

After students have predicted balance readings, they were shown actual balance readings (Figure 2b and Figure 2b).



**Figure 2a.** Twelve marbles on the electronic balance.



**Figure 2b.** Seven marbles on the electronic balance.

Students were very surprised when they saw that the observed readings differ from predicted ones. For 12 marbles, the predicted reading is bigger than actual reading (76.9488 g vs. 76.644 g), while for 7 marbles the predicted reading is smaller (44.8868 g vs. 46.320 g).

The first students' reaction was to repeat calculations because they believe strongly that when a wrong result comes out the cause is a calculation error. Only after finding that the calculations were indeed correct, students start to look for another explanations. Then they have “open ears” for the voice of those few students who recognized that calculation procedure is based on the assumption that all marbles have the same mass.

Actual weighing of marbles mass (Figure 3) shows that such an assumption, although quite natural, isn't correct. Namely, although all marbles look visually the same, their masses differ strongly. The mass difference between the first and third marble is bigger than one gram!

In addition, no marble's mass is equal to calculated "mass of one marble" (6.4124 g).



**Figure 3.** The masses of three marbles differ significantly.

## Conclusion

According the first application results reported above, this simple activity seems quite promising for introducing students into the field of critical thinking. They should always be asked to

- (1) identify the assumptions which were made; and
- (2) evaluate their correctness, either conceptually or experimentally.

The activity is also suitable to stress the importance of precise measurements in physics learning, showing that marbles, although visually seem to be of equal size, have different masses.

The results also indicate that many students do not understand that mathematical operations they use in solving physics problems imply always some assumptions about the physical objects and processes. In this case, erroneous implicit assumption was that all marbles have the same mass. In a sense, this error is related to the errors students reveal when dealing with conceptual fraction problems. Namely, when representing fractions visually, students do not consider that parts of the whole should be equal (Olive & Vomvori, 2006; Ross & Bruce, 2009).

The last, but not the least, result of informal application of the sequence is that teachers were surprised how it engaged students in meaningful discussion.

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